Benford's Law in Accounting

A Multi-Platform Data Analytics Exercise

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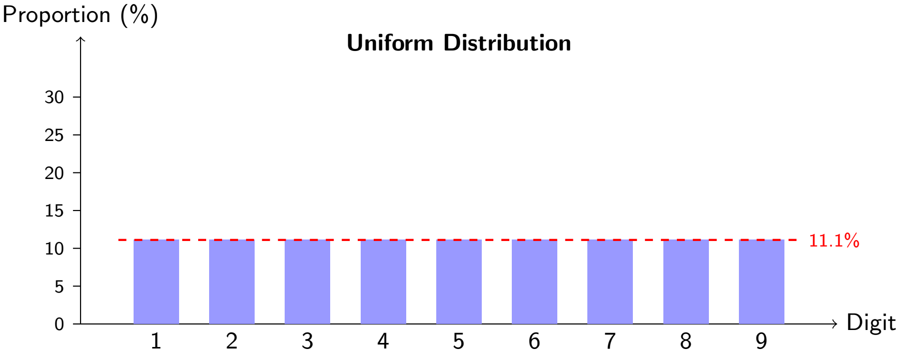
Table of Contents

Opening Question

Q. You are given a dataset of 1,000 revenue observations from companies. Think about a leading digit of each revenue observation (e.g., the leading digit of 234 is 2.) Do you expect each digit to appear equally often?

1. Yes, I expect each digit to appear equally often.
2. One (1) would be more likely observed than nine (9).
3. Nine (9) would be more likely observed than one (1).
4. I don't know.

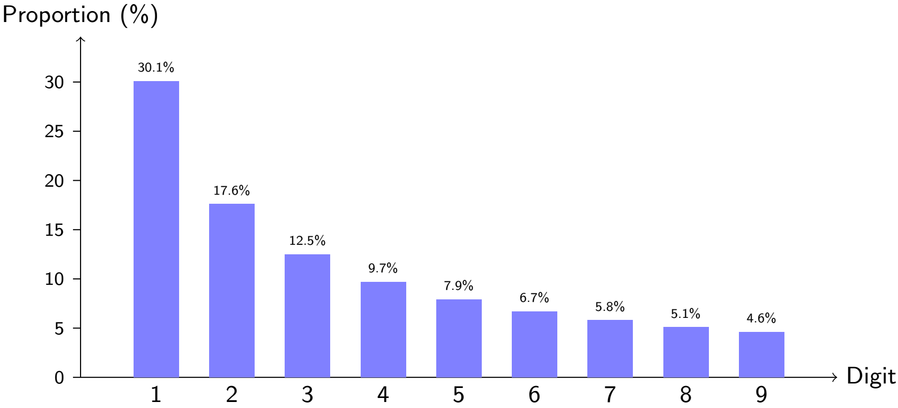
Do You Expect a Uniform Distribution?



Do you expect each digit to appear equally often?

Benford's Law

Actually, it follows **Benford's Law**!



Benford's Law Formula

$$P(D = d) = \log_{10}\left(1 + \frac{1}{d}\right), \quad d = 1, 2, \ldots, 9$$

Why Does Benford's Law Work?

- Think about a series that **grows by 20% each year**.
- Numbers spend more "time" with smaller leading digits!
- Starting at \$1, it takes a long time to go from \$1 to \$2 (leading digit 1).
- But it takes much less time to go from \$8 to \$9 (leading digit 8).
- Example:
  - $\$1 \rightarrow \$1.2 \rightarrow \$1.44 \rightarrow \$1.728 \rightarrow \$2.0736$
  - $\$8 \rightarrow \$9.6$

Benford's Law Interactive Visualization

When Does Benford's Law Apply?

Not all datasets follow Benford's Law. But many **naturally occurring** datasets do:

Applies well

- Revenue / financial data
- Population data
- Stock prices
- Tax return amounts

Does NOT apply

- Assigned numbers (SSN, zip)
- Constrained data (ATM limits)
- Narrow-range data (heights)

Table of Contents

Today's Plan

We'll do a **hands-on analysis** to see if revenue data follows Benford's Law using three tools:

1. **Microsoft Excel** — familiar, accessible
2. **Python (Google Colab)** — powerful, reproducible
3. **Claude AI** — automated, AI-assisted

Plus: a **Web Application** for generalized analysis!

Motivation: Revenue Manipulation

Key Question

What if managers have incentives to **inflate revenues** to cross round-number thresholds?

- E.g., inflating \$990K $\rightarrow$ \$1,000K
- Which number would be more observed due to the incentive?

Click here for the dynamic visualization

Our Data

- Go to Blackboard to download the data.
- The dataset includes **1,000 revenue observations** with two columns:

Obs	Revenue ($1K)	Revenue_Manipulated ($1K)
1	980,077.49	**1,058,484**
2	955,451.49	**1,031,888**
3	947,109.89	**1,022,879**
4	941,637.34	**1,016,968**
5	924,400.47	924,400.47
...	...	...
1000	1.02	1.02

- **Revenue**: natural distribution (follows Benford's Law)
- **Revenue_Manipulated**: artificially inflated near round numbers

Table of Contents

Excel: Key Functions

Function	**Purpose / Example**
`LEFT(A2,1)`	Extract first character
`VALUE(LEFT(A2,1))`	Convert to number
`COUNTIF(B:B,1)`	Count occurrences of digit 1
`SUM(C2:C10)`	Total count

Excel: Revenue Data

For **Revenue_Manipulated** data:

1. Extract the first digit using `=VALUE(LEFT(A2,1))`
2. Create a **frequency table** for each leading digit (1–9) using `COUNTIF`
3. Calculate observed proportions
4. Compare with Benford's expected proportions
5. Create a **bar chart**: observed vs. expected

	A	B
1	Revenue_Manipulated	FirstDigit
2	1,058,483,693.44	1
3	1,031,887,605.42	1
4	1,022,878,681.33	1
5	1,016,968,328.12	1
6	924,400,472.29	9
7	919,365,510.44	9
8	895,774,527.99	8
9	894,181,040.77	8
10	884,998,062.59	8

Figure: First-Digit Extraction in Excel

Excel: Revenue Data — Results

(1) FirstDigit	(2) Benford	(3) Actual Freq	(4) Actual Relative	(5) Diff	(6) AbsDiff
1	0.301	337	0.337	-0.036	0.036
2	0.176	180	0.180	-0.004	0.004
3	0.125	131	0.131	-0.006	0.006
4	0.097	106	0.106	-0.009	0.009
5	0.079	61	0.061	0.018	0.018
6	0.067	71	0.071	-0.004	0.004
7	0.058	50	0.050	0.008	0.008
8	0.051	54	0.054	-0.003	0.003
9	0.046	10	0.010	0.036	0.036
Total	1	1000	1		

Figure: Frequency Table

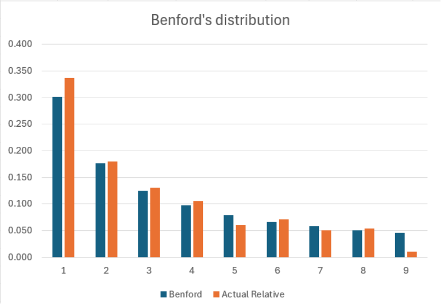


Figure: Bar Chart

Hands-On: Excel (5–10 min)

Your Turn

Now repeat the same steps for **Revenue_Manipulated**:

1. Extract the first digit: `=VALUE(LEFT(C2,1))`
2. Count each digit: `=COUNTIF(D:D,1)`, . . . , `=COUNTIF(D:D,9)`
3. Calculate observed proportions
4. Create a bar chart: observed vs. expected

Q. Compare the two charts (Revenue vs. Revenue_Manipulated). Any differences? What do you notice about digits **1** and **9**?

Table of Contents

Python: Getting Started

- Click **here to open the Google Colab notebook**
- Click the folder icon on the left sidebar.
- Drag and drop the CSV file to the area pointed by the arrow

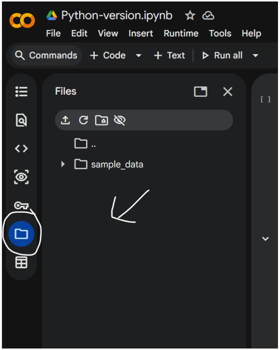


Figure: Google Colab: Where to Drop the CSV File

1. We'll run each cell one by one
2. Press `CTRL + Enter` to run each code cell

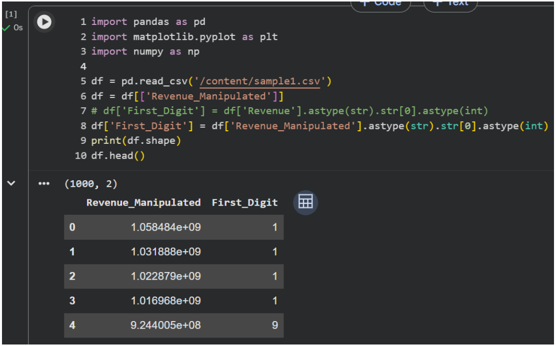


Figure: Google Colab: Data Loading

Python: Visualization

```python
# Calculate the observed frequency of the first digits
observed_counts = df['First_Digit'].value_counts().sort_index()
observed_freq = observed_counts / observed_counts.sum()

# Calculate the expected frequency based on Benford's Law
expected_freq = {d: np.log10(1 + 1/d) for d in range(1, 10)}
expected_freq_series = pd.Series(expected_freq)

# Create a DataFrame for easy plotting, with 'Expected' before 'Observed'
benford_df = pd.DataFrame({'Expected': expected_freq_series,
                           'Observed': observed_freq})

# Plot the observed and expected frequencies
benford_df.plot(kind='bar', figsize=(6, 3))

plt.title("Benford's Law Analysis of Revenue Data")
plt.xlabel("First Digit")
plt.ylabel("Frequency")
plt.xticks(rotation=0)
plt.show()
```

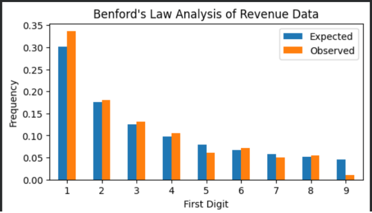


Figure: Observed vs. Expected First-Digit Distribution

Hands-On: Python (5–10 min)

Your Turn

Now repeat the same steps for **Revenue_Manipulated**:

1. Run the code cell by cell
2. Compare the results with Excel

Q. Have we got the same results as Excel?

Table of Contents

Claude AI: Getting Started

1. Go to `https://claude.ai`
2. Read the prompt below
3. Copy and paste the prompt into Claude AI
4. Drag and drop the CSV file to the Claude AI interface

Prompt

```
Analyze the first-digit distribution of \Revenue_Manipulated" column in sample1.csv
according to Benford's Law:
1.  Calculate expected proportions for digits 1{9 using Benford's Law using
\Revenue_Manipulated" column.
2.  Calculate observed proportions from the Revenues data.
3.  Create a grouped bar chart comparing expected vs.  observed proportions for each
digit.
4.  Include clear labels:  title, axis labels, and legend.
5.  Design with a clean, light theme.
6.  Make an html report file.
7.  Provide a brief interpretation using bulletpoints.
```

Claude AI: Interface

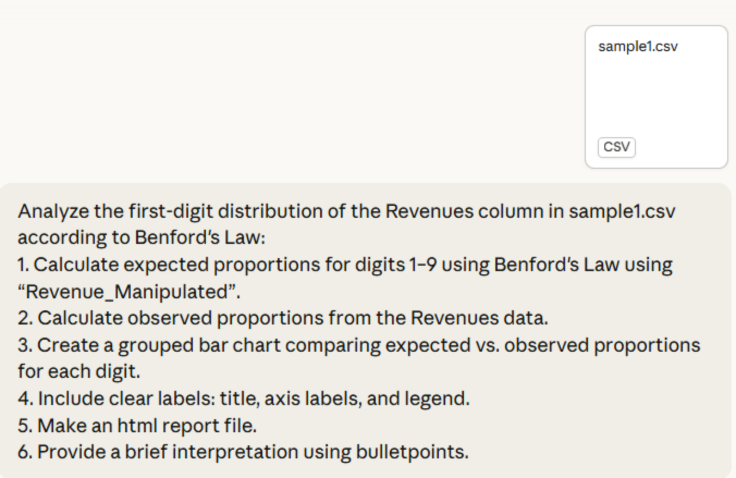


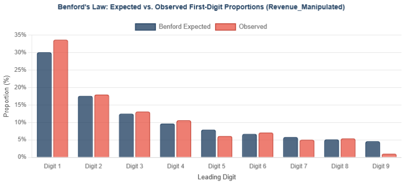
Figure: Claude AI Interface

Claude AI: Results

Benford's Law Analysis

Column: **Revenue_Manipulated** | N = 1,000 observations | Report generated February 2026

Expected vs. Observed First-Digit Distribution



Detailed Proportion Table

First Digit	Benford Expected	Observed	Difference	Deviation Flag
1	30.10%	33.70%	+3.60pp	⚠ High
2	17.61%	18.00%	+0.39pp	✓ Normal
3	12.49%	13.10%	+0.61pp	✓ Normal
4	9.69%	10.60%	+0.91pp	✓ Normal
5	7.92%	6.10%	-1.82pp	✓ Normal
6	6.69%	7.10%	+0.41pp	✓ Normal
7	5.80%	5.00%	-0.80pp	✓ Normal
8	5.12%	5.40%	+0.28pp	✓ Normal
9	4.58%	1.00%	-3.58pp	⚠ High

Figure: Visualization

Interpretation

Digit 9 severely under-represented: Observed 1.0 % vs. Benford-expected 4.6 % — a gap of −3.6 pp. This is the largest absolute deviation in the dataset and is a classic red flag for number manipulation (e.g., rounding down or truncating values before they reach the next order of magnitude).

Digit 1 slightly over-represented: Observed 33.7 % vs. expected 30.1 % (+3.6 pp). While digit 1 always dominates under Benford's Law, this excess may reflect clustering of revenue figures just above round thresholds (e.g., $1 billion+).

Digit 5 slightly under-represented: Observed 6.1 % vs. expected 7.9 % (−1.8 pp). Though moderate, combined with the digit-9 anomaly this contributes to an overall shift in the distribution toward lower digits.

Digits 2, 3, 4 broadly conform: These mid-range leading digits show deviations within ±1 pp of the Benford expectation, suggesting the bulk of the distribution is not grossly distorted.

Overall pattern is consistent with upward manipulation: The combination of elevated digit-1 frequency and sharply depressed digit-9 frequency suggests revenue figures may have been inflated just past round numbers (e.g., boosted from ~$9xx to $1,0xx million), which systematically converts 9-leading values into 1-leading values.

Recommended follow-up: Apply a chi-square goodness-of-fit test or the Kolmogorov–Smirnov test for formal statistical significance. Drill down into the digit-9 subset to identify whether specific time periods, business segments, or transaction types are driving the anomaly.

Figure: Detailed Analysis

Hands-On: Claude AI (5–10 min)

Your Turn

Now repeat the same steps for **Revenue_Manipulated** using Claude AI:

1. Read the prompt below
2. Copy and paste the prompt into Claude AI
3. Drag and drop the CSV file to the Claude AI interface

Q. Have we got the same results? Any insights?

Q. Have you generated exactly the same webpage as your peers and the instructor?

Table of Contents

Web App: Continuing from Claude AI

1. Let's continue our Claude AI conversation
2. Read the prompt below
3. Copy and paste the prompt into Claude AI

Prompt

```
Analyze the first-digit distribution of user-provided numbers according to Benford's Law:
1.  Create an input text area where users can paste numbers (one per line or
comma-separated).
2.  Add a "Go" or "Analyze" button to trigger the analysis.
3.  Display a summary section showing:  count of numbers, average value, minimum value,
and maximum value.
4.  Calculate expected proportions for digits 1{9 using Benford's Law.
5.  Calculate observed proportions from the user's data.
6.  Create a comparison table showing expected vs.  actual percentages for each digit
(1-9).
7.  Create a grouped bar chart comparing expected vs.  observed proportions for each
digit, with two bars per digit (Expected and Actual).
8.  Highlight bars with significant deviations by drawing a red dashed box around them in
the chart.
9.  Calculate and display the Chi-squared statistic and P-value.
10.  Provide a clear statistical conclusion stating whether the data likely follows
Benford's Law or not based on the P-value.  11.  Include clear labels:  title, axis
labels, and legend.
12.  Make it as an HTML interactive tool.
```

Web App: Interface

Click here to open the web app made by the instructor

Hands-On: Web App (5–10 min)

Your Turn

Now copy and paste the prompt earlier into Claude AI. You may want to modifty the prompt.

1. Once you make the webapp, test with the sample data.

Q. Have you generated exactly the same webapp as your peers and the instructor?

Table of Contents

What We've Learned

Conclusions:

- Benford's Law: a powerful tool for **detecting anomalies** in data.
- We analyzed the same data using **three platforms**.
- Cross-validation across platforms builds **confidence** in results.
- We built a **web application** for generalized analysis.
- AI tools are based on Large Language Models (LLMs).
- Even the same prompt may not always generate the same results.

Important Limitation:

- AI tools should be used **responsibly and critically**
- AI outputs may not always be correct or appropriate
- Always be **skeptical** — verify results!
- Domain expertise and professional judgment remain essential